CIVIL SERVICE EMPLOYEES ASSOCIATION

LEAP

LABOR EDUCATION ACTION PROGRAM

Examination Preparation Booklet

Basic Algebra and Elementary Statistics

Booklet No. 8

CSEA

CIVIL SERVICE EMPLOYEES ASSOCIATION, INC.
LOCAL 1000, AFSCME, AFL-CIO
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Booklet #8

Basic Algebra and Elementary Statistics

The CSEA Examination Preparation Booklet Series is designed to help members prepare for New York State and local government civil service examinations This booklet is designed for practice purposes only and its content may not conform to that of any particular civil service examination.

BASIC ALGEBRA

There is a theory that the word algebra came from the Arabic word algebrista which, in Spanish, means bone-setter. One who uses algebra, with its various devices and procedures, can "set" a problem into a shape that allows him or her to solve the problem more easily. unfortunately, for many people, algebra has hardly been this helpful. The sight of x in a word problem has been enough to trigger nightmares or bring on anger or tears. Fortunately, algebra is not needed for most promotional exams. There is, however, no doubt that, while not necessary, it is extremely useful in answering some questions on the Arithmetic Reasoning section of exams. And algebra is currently required for the Senior Statistical Clerk examination.

For these reasons, we have designed this booklet for those people who have had trouble understanding algebra, and may in fact hate it, but would like to be able to use it on promotional exams. This booklet will take you through the basic principles and procedures involved in algebra. You don't need to be a math [1]"whiz" to successfully use this booklet, although we suggest you already be comfortable with fractions, decimals and percents. (If you're not, Booklet #1 in this series, BASIC MATH, or any basic math textbook, can help).

Obviously, we'll be leaving out a lot of algebraic processes. We're including only the algebra that would be useful for promotional exams. If you find, for some strange reason, that you actually like algebra, and would like to learn more, there are many fine textbooks available.

At the end of the booklet, we [1]ve included some basic statistical work for those planning on taking the Senior Statistical Clerk examination. We've attempted to include and integrate the algebra discussed earlier with some statistical procedures you may be required to perform. While you probably won't be required to do these exact operations on the exam, this section should still be helpful, as the same algebraic principles will apply.

As is the case with all of the booklets in this series, the more effort you put into working with this booklet, the more proficient you'll become. It may not be easy at first, as many of us have had strong blocks and negative reactions to algebra for many years, but we've repeatedly seen people get past these and successfully master the material.

GOOD LUCK!

BASIC ALGEBRA

Algebra evolved from a simple and useful idea. When referring to an unknown number, it was more convenient to use a label or symbol to represent that number than to keep writing or saying "an unknown number." Mathematicians have given the unknown number a name, so that they can distinguish it from other numbers more easily. The most commonly used symbol or name for an unknown number, for our purposes, is x. But many letters of the alphabet (in Greek, Latin, italics, and lower and upper case) are used in algebra and statistics to represent or name both known and unknown numbers. Although it initially makes learning this material more difficult, once you've learned how to work with these combinations of letters and numbers, mathematical operations needed for word problems and statistics really do become easier to do. *Basic algebra is learning how to arithmetic with letters and numbers instead of just actual numbers.*

In the first part of this booklet, we're going to be outlining all the rules and procedures you'll need to do basic algebra. It may all seem strange and irrelevant, but we'll be giving you the "structural building blocks" of algebra. Without these, it will be impossible to do the algebraic and statistical problems that appear on exams. We've also included a review of working with positive and negative numbers, since this knowledge is critical. We won't be getting to working with algebraic expressions, equations or statistics until much later in the booklet. Hopefully by that time you'll have a thorough understanding of the algebraic "basics", and you'll be able to work with these much more easily. If some of this material seems familiar, please bear with us, we've designed this assuming no knowledge of algebra on the part of the reader.

SYMBOLS FOR THE FOUR FUNDAMENTAL OPERATIONS USED IN ALGEBRA

Except for multiplication, the symbols used in algebra are the same for the four basic mathematical operations as they are in regular math. It's <u>very</u> important to remember these.

Addition:	2+6 means 2 plus 6						
	x+y means x plus y						
Subtraction:	4-1 means	4 minus 1					
	x-y means	x minus y					
Division:	6÷3 means	6 divided by 3	or	$3\overline{)6}$	or	$\frac{6}{3}$	
	~.y means	x divided by y	or	$y\overline{)x}$	or	$\frac{x}{y}$	

Multiplication:	3 · 6	or	(3) (6)	or	3(6)	or 3x6	means	3 times 6
	x · y	or	(x) (y)	or	x(y)	or xy	means	x times y
	3 · x	or	(3) (x)	or	3(x)	or 3x	means	3 times x

ALGEBRAIC EXPRESS IONS

Since we re dealing with both letters and numbers in algebra, we have to be very careful to know exactly what operation we're performing when we use algebraic symbols.

For example, in using 3+x, we're adding 3 and x. If we use 3x, we're multiplying them (see above). We can also add x's: x+x+x=3x. For example, if x stands for the number 5, x+x+x would equal 5+5+5, or 15. 3x would equal 3 times 5, which is also 15. 3+x would equal 3+5, or 8. They could also be negative numbers: -x-x-x=-3x. Both 3x and -3x are examples of terms in algebra. A term is a quantity completely set off from other quantities to the left or right by either a plus or minus sign. For example, (don't get upset, it's just an example, you don[1]t need to know what it means): 3x-4yz+8z. 3x, 4yz and 8z would all be considered terms (you have to imagine there's a plus sign in front of the 3x, since it's at the beginning of the expression and not a negative number). In the term 3x, the 3 and the x are called factors. Factors are numbers that are multiplied together. The 3x is one term and has two factors: (3), and (x). 4yz is one term and has three factors (4), (y), (z). 8z is one term and has two factors (8), (z). If you had a horrible looking thing like 5a(b-c) you would still have one term. There is a minus sign, but it's inside of a parentheses, which means the result of b-c will be multiplied by 5a, so that the term is still completely set off from other possible quantities. (If you are confused, it will become clearer soon. We promise.)

The factors that make up a term are called coefficients. In the term 3x, the number 3 is the coefficient of x, and x is the coefficient of 3. 3x and x3 mean the same thing. The numerical coefficient refers to the number in a term (not the letter or letters). In 4ab, 4 is the numerical coefficient. If there is no number in front of a term, then the numerical coefficient should always be understood to be one. y means 1y, abc means labc, -x means -lx, -bc means -lbc.

Sometimes the factors which make up a term are the same. For example xxx would mean x is used three times, and would be multiplied by itself 3 times. If x equal 5x5x5. x (which is really 5 in this case), was used as a factor 3 time used as a factor four times. Because this is rather cumbersome, algebr An exponent is written slightly above and to the right of the factor that how many times the same factor is being repeated. x^3=xxx, y^4=yyyy, 5^3=(5)(5)(5), a^6=aaaaaa. Similarly $4a^3y$=4aaay, $6a^3b^4$=6aaabbbb, $3a^2b$=3aab, $12r^2s^3$=12rrsss.

PLEASE REMEMBER THE DIFFERENCE BETWEEN EXPONENTS AND NUMERICAL COEFFICIENTS.

4a = a+a+a+a a^4 = aaaa

3y = y+y+y y^3 = yyy

(a^3 would be pronounced a cubed, or a to the third power, x^2 would be pronounced x squared, or x to the second power, c^6 would be c to the sixth power, etc. Do not worry if you're not sure what all these a's, b's, r' 5, and y's represent. They merely represent unknown numbers. There's no way we could know what numbers they represent, because they're not in equations. We're just using them here to illustrate certain algebraic procedures.)

A REVIEW OF SIGNED NUMBERS

Before we continue on, it's best to briefly review positive and negative numbers, called signed numbers, and their operations. Positive numbers are numbers greater than zero. Negative numbers are numbers less than zero.

Addition of signed numbers

The sum of two positive numbers is positive. Example: +3 plus +8 equals +11.

The sum of two negative numbers is negative. Example: -7 plus -8 equals -15.

Adding numbers of unlike signs is a little more complicated, and some people have trouble with it. Try adding -24 and +16. If you're not sure, you could think of money spent as opposed to money earned. If you spent $24, and earned $16, you would have a debt of $8, or -8. WHEN ADDING NUMBERS OF UNLIKE SIGNS, YOU SHOULD FIRST SUBTRACT THE TWO NUMBERS. IF THE LARGER NUMBER IS POSITIVE, THE ANSWER WILL BE POSITIVE. IF THE LARGER NUMBER IS NEGATIVE, THE ANSWER WILL BE NEGATIVE.

If you're having a problem, try these for practice. (Answers in Answer Key on Page 37).

Add:

1) (-6)+(2)

2) (14)+(-16)

3) (-467)+(421)

4) (32)+(-13)

When adding more than two numbers, you would add up the like signs first, and then subtract. The sign of the answer will be determined by which sum of positive or negative numbers was larger. Example: (-189)+(52)+(-43)+(112). Adding -189 and -43, we get -232. Adding 52+112 equals 164. Subtracting 164 from 232, we get 68. Because the sum of the negative numbers was greater than the sum of the positive numbers, 68 will be expressed as a negative number, -68.

<u>Subtraction of Signed Numbers</u>

The rule for subtracting two signed numbers is straightforward: TO SUBTRACT TWO SIGNED NUMBERS, ADD THE OPPOSITE OF THE SECOND NUMBER. Another way to express it is to CHANGE THE SIGN OF THE NUMBER BEING SUBTRACTED AND THEN ADD.

Example: (-42)-(-33) would equal -42+33=-9. We changed the sign of the -33, because it was being subtracted, and then added the numbers together to get the answer. Some practice questions follow.

Subtract:

(45)-(-23)   You would first need to change the sign of the -23 to a +23 and then add the two numbers together.   45+23=68.

Subtract:

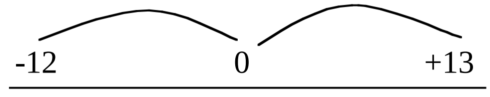
5.   -32-(-28)   6. (16)-(-24)

Example: At the beginning of the month, Alex's checking account had a total of $278.15. At the end of the month, the account was overdrawn by $43.75, excluding bank charges. If there were no deposits during the month, what was the total amount of the checks that Alex wrote?

To answer this, many of us would <u>add</u> the two amounts, $278.15 and $43.75, to get the answer, $321.90. What we could actually be doing here is subtracting the ending balance from the original balance. $278.15-(-43.75) would actually, following the method outlined above, turn into $278.l5+$43.75~ as we would have changed the sign of the number being' subtracted, and then added the numbers together. One more example to illustrate this point.

Example: When you got up at 6 a.m., the temperature was -12ºF. At noon the temperature had risen to +13º. What was the change in temperature from 6 a.m. to noon?

To find the change, we need to find the <u>difference</u> between the two temperatures, which means subtraction. We need to change the sign of the number being subtracted, and then add the numbers together, l3º-(-l2º) = 25º. (If you weren't sure which to subtract from which, it wouldn't have mattered, since all they were asking for was the <u>change</u> in temperature.) A number line could also illustrate this.



First the temperature had to go up 12º to get from -12º to 0º, then increase an additional 13º to get from 0º to 13º. Adding this distance, we get 25º.

We’ve included these examples to try and provide more context for working with positive and negative numbers.

MULTIPLICATION OF TWO SIGNED NUMBERS

When multiplying two numbers: IF THE NUMBERS HAVE THE SAME SIGN, THE PRODUCT IS POSITIVE.

Examples: (3) (4)=12 (-4) (-6)=24.

IT'S IMPORTANT TO REMEMBER THAT THE PRODUCT OF TWO NEGATIVE NUMBERS IS POSITIVE. IF THE NUMBERS HAVE DIFFERENT SIGNS, THE PRODUCT IS NEGATIVE.

Examples: (-3) (2)=-6 (17) (-2)=-34

Multiply:

7. (-8) (-18)

8. (14) (-7)

DIVISION OF TWO SIGNED NUMBERS

Division of two positive and negative numbers is similar to multiplication. THE QUOTIENT (RESULT) OF TWO NEGATIVE NUMBERS IS POSITIVE. THE QUOTIENT OF A POSITIVE AND A NEGATIVE NUMBER IS NEGATIVE.

Example: (-8)-:(4)=-2

Divide:

9. (144)-:(-12)

10. (-221)-:(-17)

MULTIPLYING AND DIVIDING STRINGS OF NUMBERS

TO MULTIPLY OR DIVIDE STRINGS OF NUMBERS, ONE SHOULD FIRST MULTIPLY AND DIVIDE AS IF THE NUMBERS WERE UNSIGNED. Then, IF THERE IS AN EVEN NUMBER OF MINUS SIGNS, THE RESULT IS POSITIVE. IF THERE IS AN ODD NUMBER OF MINUS SIGNS, THE RESULT IS NEGATIVE.

Example: (5) (-3) (-2) (-6)=-180. There are three minus signs, an uneven number, so the result will be negative.

Example: (6) (-2) (-7) (-4) (-3)=1008. There are four minus signs, an even number, so the result will be positive.

If you have an exponent, for example 2^4, remember that this represents 2 multiplie four times. 2x2x2x2=16. If it was a $(-2)^4$, BECAUSE THE EXPONENT FOUR IS AN EVEN NUMBER, THERE WOULD BE AN EVEN NUMBER OF MINUS SIGNS IN THE PRODUCT, AND THE ANSWER WOULD BE EVEN. $(-2)^4$ = l6. IF IT WAS $(-2)^3$ BECAUSE THE EXPONENT THREE IS AN ODD NUMBER, THERE WOULD BE AN ODD NUMBER OF MINUS SIGNS IN THE PRODUCT, AND THE ANSWER WOULD BE ODD. (-2) (-2) (-2) = -8.

This review of signed numbers was necessary before we could continue on with algebraic expressions.

BACK TO ALGEBRAIC EXPRESSIONS

Earlier, a term and factors were explained. The terms of an algebraic expression are separated by + and - symbols. The expression 4a+2b+c has three terms, 4a, 2b, and c. The expression 4a has two factors, (4) and (a). In algebra, it's critical to know the difference between a term and a factor, as we'll soon see.

Addition in Algebraic Expressions

In algebra as well as in arithmetic, you can only add quantities that are exactly the same kinds of things (the old apples and oranges principle). 2 cats + 3 cats = 5 cats, but 2 cats and 3 gorillas equals 2 cats and 3 gorillas (the old cats and gorillas principle). In math, we have to find common denominators before we can add fractions together. That is because we can only add like things. The same is true in algebra.

Example: $\frac{3}{8} + \frac{2}{6}$ equals $\frac{3}{8} + \frac{2}{6}$ (we can't combine them because they aren't the same).

$\frac{3}{8} + \frac{2}{8}$ equals $\frac{5}{8}$ (we can combine them because they are the same).

In algebra:

3a+2a equals 5a (we can combine them)

but $3a^2 + 2a$ equals $3a^2 + 2a$ (we can't combine them, as they are different. a^2 and a's are not the same).

4ab+6ab equals l0ab, but $4ab+6ab^2$ equals $4ab+6ab^2$ (again, we can't combine them because they are not alike; ab and ab^2's are different).

Try adding:

$4x-2y$, $2x^2-6y$, $l0x-2y^2$

At this point you may remember exactly why you hated algebra so much. Actually, it's not as bad as it looks. You need to combine like terms. We can only combine terms that are exactly alike. When terms are exactly alike, except that their numerical coefficients may be different (see beginning of ALGEBRAIC EXPRESSIONS above), we can add them simply by adding their numerical coefficients. Looking at the above mess, we need to determine which terms are alike. If we take the time and examine it carefully, we'll be able to break it down in the following manner: there are x's (a 4x and a l0x), y's (a -2y and -6y), an x^2 ($2x^2$) and a y^2 ($-2y^2$).

First we can combine the x's. 4x+l0x=14x. Then the y's: -2y+(-6y)=-8y. There's only one x^2 and one y^2, so there's no combining necessary there.

So, $4x-2y$, $2x^2-6y$, $l0x-2y^2$ will become, by adding the like quantities, $14x-8y+2x^2-2y^2$.

Add: $4a^2+4a+17$ and $8a^2-7a-21$.

The a^2's are $4a^2$ and $8a^2$, the a's are 4a and -7a, and the numbers are 17 and -21. Combining, we'll get $12a^2-3a-4$.

Add: $12r^2+16r^2+3r^2$.

Since these are all the same terms, we can just add them together, getting $31r^2$.

Add: $7m^2-4m^2n^2+3n^2$ and $-11m^2n^2+8n^2-3m^2$.

Adding $7m^2$ and $-3m^2$ equals $4m^2$. Adding $-4m^2n^2$ and $-11m^2n^2$ equals $-15m^2n^2$.

Adding $3n^2$ and $8n^2$ equals $11n^2$. The answer is: $4m^2-15m^2n^2+11n^2$.

Adding Algebraic Expressions Which Are Fractions

To do this, you would proceed just as if you were adding regular fractions. If the denominators are different, you would need to find a common denominator.

Example: Add $\frac{2}{a}$ $\frac{4}{b}$

The common denominator is ab. $\frac{2}{a} = \frac{2b}{ab}$, and $\frac{4}{b} = \frac{4a}{ab}$

(Some people say to themselves, "a goes into ab once, with b left over. So 2 times b = 2b, for the numerator". For the second fraction, .[11]b goes into ab once, with a left over. So 4 times a = 4a for the numerator.")

So, $\frac{2}{a} \quad \frac{4}{b} \qquad \frac{2b}{ab} \quad \frac{4a}{ab} \_ \quad \frac{2b + 4a}{ab}$

Don't worry if this isn't perfectly clear, it's unlikely you would need it.

Subtraction in Algebraic Expressions

Subtracting in algebraic expressions gets a little tricky. TO SUBTRACT, WE MUST REVERSE THE SIGN OF THE SUBTRAHEND (THE NUMBER BEING SUBTRACTED), AND THEN FOLLOW THE RULES FOR ADDITION.

Example: Subtract x^2-14x+6 from $4x^2$+7x-8.

The subtrahend is x^2-14x+6 (it's being subtracted). We first need to reverse the sign of the subtrahend. This means each term in the subtrahend will have its sign reversed. x^2 becomes $-x^2$ [1] -14x becomes +14x and +6 becomes -6. So, the subtrahend reversed is: $-x^2$+14x-6. We've reversed the signs. The next thing we need to do is then follow the rules for addition. So, after reversing the signs, we'll add. Adding $-x^2$+14x-6 plus $4x^2$+7x-8, we get $3x^2$+21x-14. (At this point, some of you may still be confused about the difference between an x^2 or x^3 and something like 2x or 3x. x^2 and x^3 are exponents, which means whatever number x represents is being multiplied by itself two or three times, respectively. If x equaled 3, then x^2 would equal 3x3, or 9. x^3 would equal 3x3x3, or 27. In the case of 2x, however, the expression means that whatever number x is, the number we are looking for would be two times x. If x were 3, 2x would be 2x3, or 6. 3x would equal three times three, or nine.)

Subtract -4ab+$6a^2$+$4b^2$ from $2a^2b^2$+6ab-$3b^2$.

The subtrahend is -4ab+$6a^2$+$4b^2$. Reversing signs it becomes +4ab-$6a^2$-$4b^2$. Adding them together, we get $2a^2b^2$+10ab-$6a^2$-$7b^2$.

ANOTHER IMPORTANT RULE TO REMEMBER IN' SUBTRACTING ALGEBRAIC EXPRESSIONS IS THE RULE FOR REMOVING PARENTHESES. IF THE SIGN JUST BEFORE THE PARENTHESES IS POSITIVE, SIMPLY REMOVE THE PARENTHESES. IF THE SIGN JUST BEFORE THE PARENTHESES IS NEGATIVE, CHANGE THE SIGN OF EVERY EXPRESSION INSIDE THE PARENTHESES, AND THEN REMOVE THE PARENTHESES.

Example: Combine (or simplify) a-(-2a+3).

Since there is a minus sign just before the parentheses, we'll need to change the signs of the expressions inside the parentheses. (-2a+3) becomes +2a-3.

a-(-2a+3) becomes a+2a-3, which equals 3a-3.

Simplify: (2a+3b)- (a-6b).

(2a+3b) becomes 2a+3b (we can just remove the parentheses, since there is no minus sign in front of it). (a-6b) becomes -a+6b~ since there is a minus sign in front of the parentheses. Simplifying, we combine 2a+3b and -a+6b, and get: a+9b.

Multiplying Algebraic Expressions

In algebra, it's critical that you be able to distinguish a factor from a term. Again, a term is a quantity completely set off from other quantities (to the left or right) by either a plus or a minus sign. One term may contain many factors, and a factor may contain more than one term.

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Example: $6a^2+9b-4a^3b^2$ The terms here are $6a^2$, 9b and -4a b

Factors are quantities which are multiplied together.

Example: $9x^2$+l3xyz-$17y^3$

$9x^2$ is one term.
The 2 factors are 9 and x^2, but since factors are any quantities which are multiplied together, $9x^2$ is also a factor.

l3xyz is one term.
The 4 factors are 13, x, y and z, and l3xyz is also a factor.

$-17y^3$ is one term.
The 2 factors are -17, and (y^3), and $-17y^3$ is itself also a factor.

Remember, since factors may contain more than one term, and factors are quantities which are multiplied together, the following are also factors:

$(3x^2+2y)\ (3y)$

Both $(3x^2+2y)$ and (3y) are factors, because they are quantities that are multiplied together.

IN ALGEBRA, TO MULTIPLY ANY TWO FACTORS, WE MUST MULTIPLY EACH TERM IN THE FIRST FACTOR SEPARATELY BY EACH TERM IN THE SECOND FACTOR. IF THERE ARE ANY SIMILAR TERMS RESULTING, WE WILL THEN COMBINE THEM.

Examples: (2a) (3b) = 6ab
$(-3)\ (-6x^2y^2) = 18x^2y^2$

Another way to state this is to perform the numerical and letter multiplication separately, and then multiply the results.

This is all an expression of the distributive law, which states $a(b+c) = ab + ac$. And $a(b-c) = ab-ac$. (This will all become clearer soon).

While this multiplication holds for all types of algebraic expressions, there is one exception. When we have repeated factors multiplied together, the exponents of the repeated factors are actually added, not multiplied.

Examples: $(a^3)(a^4) = (aaa)(aaaa)$
$= a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a$
$= aaaaaaa$
$= a^7$

or $(a^3)(a^4) = a^{3+4} = a^7$, not a^{12}

$(x^5)(x^2) = x^7$

$(y^3)(y^3) = y^6$

This is all very confusing for many people, so we've included these examples.

Example: $2a(a+b) = 2a(a) + 2a(b)$
$= 2a^2+2ab$

We're multiplying each term in the first factor separately by each term in the second factor and then combining similar terms, when possible.

Or, if it's easier, we're doing the numerical and letter multiplication separately, and then multiplying the results.

Some people don't like using either of these. Just by studying enough examples, they get the "feel" of it. So we'll give a lot of examples.

Examples

$-4b(3a-6b) = -4b(3a)-4b(-6b)$
$= -12ab+24b^2$ (the product of two negative numbers is positive)

$y(4x^2 +2y) = 4x^2(y)+2y(y)$
$= 4x^2y+2y^2$

$a(b+c+d+e) = ab+ac+ad+ae$

$-4x(2x+4y-z+2) = -8x^2-16xy+4xz-8x$

-

$2a^2(4a^2-3ab+c^3) = 8a^4-6a^3b+2a^2c^3$

$2x^2(3x^2-4x+3) - 6x^4-8x^3+6x^2$

$2x^2yz(x^{2}\sim y^2+3z^2) = 2x^4yz-2x^2y^3z+6x^2yz^3$

$4a^3(a^3+b^2+c) = 4a^6+4a^3b^2+4a^3c$

If you re still a little confused, try going back over this section several times, it should help. You shouldn't continue on until you feel comfortable with the above examples and principles.

So far, we've multiplied things like (4a) (-6b) = -24ab (called multiplying a monomial by a monomial).

We[1]ve also multiplied a monomial times a multinomial (an expression which contains more than one term). $(4a)(4a^3-3b+a) = 16a^4-l2ab+4a^2$.

Now, we're going to multiply a multinomial by a multinomial. This is the most difficult multiplication of these algebraic expressions. However, if you remember the distributive law, and <u>always multiply each term in one factor separately by each term in the other factor</u>, you'll be able to do this with little trouble.

When multiplying multinomials by multinomials, another way to express the distributive law is:

$$(a+b)(c+d) = a(c+d) + b(c+d)$$

$$= ac + ad + bc + bd$$

Using this as a guide, we can multiply multinomials by multinomials. We'll give lots of examples.

$$(x+y)(x+y) = x(x+y) + y(x+y)$$

$$= x(x) + x(y) + y(x) + y(y)$$

$$= x^2 + xy + yx + y^2$$

You may remember that xy would be identical to yx so they can be combined. xy = yx. xy + xy = 2xy. So,

$(x+y)(x+y) = x^2 + 2xy + y^2$.

$(a+b)(a-b) = a(a-b) + b(a-b)$

$= a(a) + a(-b) + b(a) + b(-b)$

$= a^2 - ab + ba - b^2$

-ab and ba are identical, and cancel each other out. So:

$(a+b)(a-b) = a^2 - ab + ab - b^2 - a^2 b^2$

$(2a+2\}(a-2) = 2a(a-2) + 2(a-2)$

$= 2a(a) + 2a(-2) + 2(a) + 2(-2)$

$= 2a2 - 4a + 2a - 4$

$= 2a^2-2a-4$

$(x+y)(x^2-xy+y^2) = x(x^2-xy+y^2) + y(x^2-xy+y^2)$

$= x(x^2) + x(-xy) + x(y2) + y(x^2) + y(-xy) + y(y^2)$

$= x^3 + -x^2y + xy^2 + yx^2 - xy^2 + y^3$

$-x^2y$ and yx^2 cancel each other out, as do xy^2 and $-xy^2$. So, $(x+y)(x^2-xy+y^2) - x^3+y^3$

$(6r-s)(2r+3s) = 6r(2r+3s) - s(2r+3s)$

$= 6r(2r) + 6r(3s) - s(2r) - s(3s)$

$= 12r2 + l8rs - 2sr - 3s^2$

$= 12r^2 + l6rs - 3\sim2$

$(a+b)^2 = (a+b)(a+b) = a(a+b) + b(a+b)$

$= a(a) + a(b) + b(a) + b(b)$

$= a^2 + ab + ba + b^2$

$= a^2+2ab+b^2$

$(6x-3)^2 = (6x-3)\ (6x-3) = 6x(6x-3) - 3(6x-3)$

$= 6x(6x) + 6x(-3) - 3(6x) - 3(-3)$

$= 36x^2 -18x -18x +9$

$= 36x^2-36x+9$

$(x+y)^3 = (x+y)\ (x+y)\ (x+y)$

$= (x^2 + xy + yx + y^2)\ (x+y)$

$= (x^2 + 2xy + y^2)\ (x+y)$

$= x^2(x+y) + 2xy(x+y) + y^2(x+y)$

$= x^3 + x^2y + 2x^2y + 2xy^2 + y^2x + y3$

$= x^3+3x^2y+3xy^2+y^3$

$(2x+5)\ (x+3) = 2x(x+3) + 5(x+3)$

$= 2x(x) + 2x(3) + 5(x) + 5(3)$

$= 2x^2 + 6x + 5x + 15$

$= 2x^2+llx+15$

Dividing Algebraic Expressions

Dividing Monomials

Dividing monomials in simple algebraic expressions is like reducing a fraction. We divide the numerator and denominator by the factors they have in common. For example, in arithmetic, we would simplify a fraction by reducing it to its lowest terms.

$$\frac{30}{9} = \frac{(3)\ (10)}{(3)\ (3)} = \frac{(\overset{1}{\cancel{3}})\ (10)}{(3)\ (\underset{1}{\cancel{3}})} = \frac{l \times l0}{3x1} = \frac{10}{3} = 3\ 1/3$$

$$\frac{6x}{3x} = \frac{(3)\ (2)\ (x)}{(3)\ (x)} = \frac{(\overset{1}{\cancel{3}})\ (2)\ (\overset{1}{\cancel{x}})}{(\underset{1}{\cancel{3}})\ (\underset{1}{\cancel{x}})} = \frac{l \times 2 \times l}{l \times l} = \frac{2}{1} = 2$$

$$\frac{8r^2}{4r} = \frac{(4)(2)(r)(r)}{(4)\ (r)\ (4)} = \frac{(\overset{1}{\cancel{4}})(2)(\overset{1}{\cancel{r}})(r)}{(\underset{1}{\cancel{4}})\ (\underset{1}{\cancel{r}})} = \frac{l \text{ x } 2 \text{ x } l \text{ x } r}{l\text{x}l} = \frac{2r}{1} = 2r$$

$$\frac{7m}{7mn} = \frac{(7)(m)}{(7)(m)(n)} = \frac{(\overset{1}{\cancel{7}})(\overset{1}{\cancel{m}})}{(\underset{1}{\cancel{7}})(\underset{1}{\cancel{m}})(n)} = \frac{l \text{ x } l}{l \text{ x } l \text{ x } n} = \frac{1}{n}$$

Divide $20a^2b^2$ by $4ab$.

$$\frac{20a^2b^2}{4ab} = \frac{(5)(4)(a)(a)(b)(b)}{(4)(a)(b)} = \frac{(5)(\overset{1}{\cancel{4}})(\overset{1}{\cancel{a}})(a)(\overset{1}{\cancel{b}})(b)}{(\underset{1}{\cancel{4}})(\underset{1}{\cancel{a}})(\underset{1}{\cancel{b}})} = \frac{5 \text{ x } l \text{ x } l \text{ x } a \text{ x } l \text{ x } b}{l \text{ x } l \text{ x } l}$$

$$\frac{5 \text{ x } a \text{ x } b}{1} = \frac{5ab}{1} = 5ab$$

Divide $18m^4n^3$ by $3mn$.

$$\frac{18m^4n^3}{3mn} = \frac{(6)(3)(m)(m)(m)(m)(n)(n)(n)}{(3)(m)(n)} = \frac{(6)(\overset{1}{\cancel{3}})(\overset{1}{\cancel{m}})(m)(m)(m)(\overset{1}{\cancel{n}})(n)(n)}{(\underset{1}{\cancel{3}})(\underset{1}{\cancel{m}})(\underset{1}{\cancel{n}})}$$

$$\frac{6 \text{ x } l \text{ x } l \text{ x } m \text{ x } m \text{ x } m \text{ x } i \text{ x } n \text{ x } n}{1 \text{ x } 1 \text{ x } 1} = \frac{6\text{x}m^3\text{x}n^2}{1} = \frac{6m^3n^2}{1} = 6m^3n^2$$

Divide $21m^2n^3$ by $14m^2n^6$

$$\frac{21m2n3}{l4m^2n^6} = \frac{(7)(3)(m)(m)(n)(n)(n)}{(7)(2)(m)(m)(n)(n)(n)(n)(n)(n)} = \frac{(\overset{1}{\cancel{7}})(3)(\overset{1}{\cancel{m}})(\overset{1}{\cancel{m}})(\overset{1}{\cancel{n}})(\overset{1}{\cancel{n}})(\overset{1}{\cancel{n}})}{(\underset{1}{\cancel{7}})(2)(\underset{1}{\cancel{m}})(\underset{1}{\cancel{m}})(\underset{1}{\cancel{n}})(\underset{1}{\cancel{n}})(\underset{1}{\cancel{n}})(n)(n)(n)}$$

$$\frac{l \text{ x } 3 \text{ x } l \text{ x } l \text{ x } l \text{ x } l \text{ x } l}{l \text{ x } 2 \text{ x } l \text{ x } l \text{ x } l \text{ x } l \text{ x } l \text{ x } n \text{ x } n \text{ x } n} = \frac{3}{2n^3}$$

Of course you don't have to go through the lengthy procedure described above, we're just trying to help you visualize what is going on with these types of questions. You can achieve the same results by dividing the numerical and letter parts of the expressions separately.

For example $\frac{44r^3s}{llrs}$ could also be done by first dividing the numbers,

then the letters, $44 \div 11 = 4$, $r^3 \div r = r^2$, and $s \div s = 1$, so $44r^3s \div l1rs$ would equal $4r^2$. If you feel more comfortable writing it out and then dividing, or canceling, that's fine.

$$\frac{44r^3s}{11rs} = \frac{(4)(11)(r)(r)(r)(s)}{(11)(r)(s)} = \frac{(4)(\overset{1}{\cancel{11}})(\overset{1}{\cancel{r}})(r)(r)(\overset{1}{\cancel{s}})}{(\underset{1}{\cancel{11}})(\underset{1}{\cancel{r}})(\underset{1}{\cancel{s}})} = \frac{4 \times r^2}{1} = 4r^2$$

Notice that we've been saying to divide the <u>factors</u> they have in common.
THIS APPLIES TO FACTORS ONLY, NEVER TO TERMS. IN ALGEBRA, YOU CAN DIVIDE OUT (OR CANCEL) THE COMMON FACTORS, BUT NEVER CANCEL TERMS.

For example, you could divide out (or cancel) the factors in the following:

$$\frac{12a^3b}{-3ab} = \frac{(4)(3)(a)(a)(a)(b)}{(-3)(a)(b)(b)(b)} = \frac{(4)(\overset{-l}{\cancel{3}})(\overset{1}{\cancel{a}})(a)(a)(\overset{1}{\cancel{b}})}{(-\underset{1}{\cancel{3}})(\underset{1}{\cancel{a}})(b)(b)(\underset{1}{\cancel{b}})} = \frac{-4a^2}{b^2} \text{ or } -\frac{4a^2}{b^2}$$

You could <u>not</u> divide or cancel out the following because you cannot divide out or cancel a <u>term</u>. (Again, terms are set off by + and - signs, factors are quantities which are multiplied together).
Examples of terms:

$$\frac{x^2 +\sim 2}{x^3} \quad \text{or} \quad \frac{a + b^3}{b^2}$$

If you have trouble remembering this, you can think of examples using "real" numbers. There's a big difference between the following fractions, and what you can do with them.

$$\frac{(7)(3)}{(3)} \quad \text{or} \quad \frac{7 + 3}{3}$$

You can divide (or cancel) in the first example, because the factors are being multipled. So,

$$\frac{(7)(\overset{1}{\cancel{3}})}{(\underset{1}{\cancel{3}})} = \frac{7}{1} = 7 \qquad \text{Or:} \qquad \frac{(7)(3)}{(3)} = \frac{21 \;\; = \;\; 7}{3}$$

The second fraction means $\frac{7 + 3}{3} = \frac{10}{3}.$

Since 7 + 3 are being added, not multipled, they are not factors, and cannot be divided out or factored.

Review of Exponents

Remember, exponents are added or subtracted from each other, not multipled or divided. So $(a^6)(a^4)$ would equal a^{10}, not a^{24}. And $a^{10} \div a^5$ would equal a^5, not a^2. If the divisor is larger than the dividend (the number being divided), then the result is expressed in the following way:

$$a^4 \div a^8 = \frac{aaaa}{aaaaaaaa} = \frac{\overset{1111}{\cancel{aaaa}}}{\underset{1111}{\cancel{aaaa}}aaaa} = \frac{1}{a^4}.$$

$$\frac{x^3}{x^6} = \frac{\overset{111}{\cancel{xxx}}}{\underset{111}{\cancel{xxx}}xxx} = \frac{1}{x^3}.$$

Dividing a Multinomial by a Monomial

This is a little more complicated, but you will always get the right answer if you always DIVIDE EACH SEPARATE TERM OF THE DIVIDEND (THE NUMBER BEING DIVIDED) BY THE ENTIRE DIVISOR.

Examples: $$\frac{x + y}{x} = \frac{x}{x} + \frac{y}{x} = 1 + \frac{y}{x}$$

$$\frac{a^4 - 3a^3}{a} = \frac{a^4}{a} - \frac{3a^3}{a} = a^3 - 3a^2$$

In each case, we've divided each term of the dividend by the divisor.

Example:
$$\frac{4x^4 - 8x^2 - 12x}{4x} = \frac{4x^4}{4x} - \frac{8x^2}{4x} - \frac{12x}{4x} = x^3-2x-3$$

Example: $\dfrac{2a^2b - 8ab^2 + 4ab}{6a^2b^2}$

$$\frac{2a^2b - 8ab^2 + 4ab}{6a^2b^2} = \frac{2a^2b}{6a^2b^2} - \frac{8ab^2}{6a^2b^2} + \frac{4ab}{6a^2b^2} = \frac{1}{3b} - \frac{4}{3a} + \frac{2}{3ab}$$

Or you could do it like this:

$$\frac{2a^2b - 8ab^2 + 4ab}{6a^2b^2} = \frac{2a^2b}{6a^2b^2} - \frac{8ab^2}{6a^2b^2} + \frac{4ab}{6a^2b^2}$$

$$= \frac{(2)(a)(a)(b)}{(2)(3)(a)(a)(b)(b)} - \frac{(4)(2)(a)(b)(b)}{(3)(2)(a)(a)(b)(b)} + \frac{(2)(2)(a)(b)}{(3)(2)(a)(a)(b)(b)}$$

$$= \frac{\overset{1}{\cancel{(2)}}\overset{1}{\cancel{(a)}}\overset{1}{\cancel{(a)}}\overset{1}{\cancel{(b)}}}{\underset{1}{\cancel{(2)}}(3)\underset{1}{\cancel{(a)}}\underset{1}{\cancel{(a)}}\underset{1}{\cancel{(b)}}(b)} - \frac{(4)\overset{1}{\cancel{(2)}}\overset{1}{\cancel{(a)}}\overset{1}{\cancel{(b)}}\overset{1}{\cancel{(b)}}}{(3)\underset{1}{\cancel{(2)}}\underset{1}{\cancel{(a)}}(a)\underset{1}{\cancel{(b)}}\underset{1}{\cancel{(b)}}} + \frac{\overset{1}{\cancel{(2)}}(2)\overset{1}{\cancel{(a)}}\overset{1}{\cancel{(b)}}}{(3)\underset{1}{\cancel{(2)}}\underset{1}{\cancel{(a)}}(a)\underset{1}{\cancel{(b)}}(b)}$$

$$= \frac{1}{3b} - \frac{4}{3a} + \frac{2}{3ab}$$

Example: $\dfrac{14a^2b^2 - 21a - 7ab}{-7ab} = \dfrac{14a^2b^2}{-7ab} - \dfrac{21a}{-7ab} - \dfrac{7ab}{-7ab} = -2ab + \dfrac{3}{b} + 1$

or: $$\frac{(7)(2)(a)(a)(b)(b)}{(-7)(a)(b)} - \frac{(7)(3)(a)}{(-7)(a)(b)} - \frac{(7)(a)(b)}{(-7)(a)(b)}$$

$$= \frac{\overset{1}{\cancel{(7)}}(2)\overset{1}{\cancel{(a)}}(a)\overset{1}{\cancel{(b)}}(b)}{\underset{-1}{\cancel{(-7)}}\underset{1}{\cancel{(a)}}\underset{1}{\cancel{(b)}}} - \frac{\overset{1}{\cancel{(7)}}(3)\overset{1}{\cancel{(a)}}}{\underset{-1}{\cancel{(-7)}}\underset{1}{\cancel{(a)}}(b)} - \frac{\overset{1}{\cancel{(7)}}\overset{1}{\cancel{(a)}}\overset{1}{\cancel{(b)}}}{\underset{-1}{\cancel{(-7)}}\underset{1}{\cancel{(a)}}\underset{1}{\cancel{(b)}}} = -2ab + \frac{3}{b} + 1$$

(If you're confused about what to do with positive and negative numbers, please review the section on signed numbers).

Try these:

11. $\dfrac{4m^2 - 8m}{2m}$

12. $\dfrac{14xy - 3x^2y^2}{7xy^3}$

It may be helpful to remember the distributive law,

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \quad \text{and} \quad \frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$$

This is all of the division of algebraic expressions that has been required for our exam purposes. Dividing multinomials, special products and factoring will not be included here. If you'd like to learn how to do these, there are many algebra texts available.

There is one more topic we need to cover before we examine types of possible exam questions.

The Order of Algebraic Operations

In algebra, it's necessary to determine the order of operations when we have more than one operation in a problem. For instance, 40 ÷ 2 - 24 ÷ 3. We need to know when to do what, and the following rules should be followed. In an exercise with more than one operation, you should use the following rules: (They apply for arithmetic as well).

1. Do what is inside the parentheses first. (Parentheses are also implied below and above any fraction bar).

2. Do exponents next.

3. Working from left to right do multiplications and divisions as you come to them.

4. Go back to the left and work to the right doing additions and subtractions.

So, it's: Parentheses
Exponents
Multiplication
Division
Addition
Subtraction

This is important to remember, and you may recall a device that can help. Using the first letter of each operation, you can memorize the order by thinking "Flea5e Excuse My Dear Aunt Sally." If you want something a little different, you could use "Please Excuse My Dreadful Algebraic Skills," or "Please Encourage My Daring Algebraic Skills," depending upon your outlook. Or make up your own. Using just numbers, let's use these rules.

$8(3 \cdot 4-6) \div 6 - 3$

We need to first deal with what is inside the parentheses. (3~4-6) would equal (12-6), or 6. There are no exponents, so we would proceed and multiply and divide from left to right, as we came to each operation.

$8(12-6) \div 6 - 3$
$8(6) \div 6 - 3$
$48 \div 6 - 3$
$8 - 3$
$= 5$

It is only after we're done multiplying and dividing that we then add or subtract.

$40 \div 5 - 2 + 20 \div 10 + 2 =$

$8 - 2 + 2 + 2 = 10$

It's also important to remember the rule for the removal of parentheses in algebraic expressions discussed earlier. If the sign just before the parentheses is positive[1] just remove the parentheses. If the sign just before the parentheses is negative, change the sign of every expression inside the parentheses and then remove the parentheses.

Example: Simplify $2x - 3(x+y)$
$2x - 3(x+y)$ $2x - 3x - 3y$ $-x - 3y$

Example: Simplify $(a+2b) - (a-4b)$
$(a+2b) - (a-4b) = a + 2b - a + 4b = 6b$

Example: Simplify $(x+3y) - (-2x+4y)$
$(x+3y) - (-2x+4y)$ $x + 3y + 2x - 4y = 3x - y$

EVALUATING ALGEBRAIC EXPRESSIONS

We've tried to provide you with some basic tools for understanding and working with algebraic expressions. Now we're going to apply some of these in answering the types of questions that could appear on promotional examinations.

Sometimes people are required to convert words into algebraic expressions. For example:

The best representation of seven times a number plus eight would be:

a. $7x^2 8y^2$ c. x+7+x+8
b. 7x+8 d. (7n) (8)

Remember that in algebra, unknown numbers are represented by letters. An unknown number can be represented by any letter, so it doesn't matter what letter is used. To successfully answer the above question, you need to have set up the proper relationship between the unknown number and the wording in the problem. In these types of problems, always substitute a letter (any letter will do) for the words "a number", and proceed from there. In the above question, you could substitute as you read along. "The best representation of 7 times a number" would be 7x, "plus eight" would be 7x+8, choice b. What if they didn't have x's? It wouldn't have mattered. The possible answer could have been 7n+8, 7r+8, 7s+8, etc. You can use <u>any</u> letter to represent an unknown number. Often on tests they'll use any of the 26 letters of the alphabet. That doesn't matter. What matters is setting up the correct relationship between the numbers and letters.

Example: How would one represent three times a number less two?

a. 3x(+2) c. 3k-2
b. 2r-3 d. -2-3x

If you substituted a letter for the phrase "a number" while you did this, you would get 3n-2, or 3k-2, choice c.

The best representation of "four less than one-half a number" would be:

a. 1/2r-4 c. $\frac{x+4}{2}$
b. 1/2x+4 d. $\frac{x}{2} - \frac{4}{2}$

The answer is a. If we write it out, substituting a letter for the phrase a number," we'll get four less than 1/2x, which equals 1/2x-4; choice a. (Again, any letter can be selected to represent an unknown number. On an exam, you would select the choice that correctly expresses the relationship between the unknown number and the other numbers.)

Write each of the following as an algebraic expression, letting m represent the unknown. The answers are in the Answer Key.

13. "three times a number plus 17"
14. "Subtract four times a number from nineteen."
15. "a number divided by eight"
16. "fourteen less than a number plus three times the number"

Write each of the following as an algebraic expression. Let r represent the first unknown and s represent the second unknown.

17. "four times the first number less seven times the second one"

18. "the product of the two numbers plus three times the first one"
19. "the square of the first number plus half of the second"
20. "two times the square of the second number and 1/3 of the first number"

Another sample question:

Represent the amount by which 56 exceeds 4 times a number.

a. $4x = 56$ c. 56-4k
b. $56 + 4r(2)$ d. $4r + 56$

This is a little tricky. We're not trying to solve anything, we' re looking for the amount by which 56 exceeds 4 times a number. The number should be represented by a letter. They chose k. Then 4 times the number is 4k. 56 exceeds 4k by the amount 56 - whatever 4k is. So the answer would be 56-4k. It may help, if you have trouble visualizing this, and a lot of people do, to work with "real numbers," and substitute in a real number in order to see the relationship more clearly. Let's pretend k = 4. We're trying to find the amount by which 56 exceeds 4 times k. If we substitute in a real number for k, perhaps this will become easier. If we pretend k equals 4, then 4k would equal 4 x 4, or 16. To find (or represent) the amount by which 56 exceeds 4k, or the amount 56 exceeds 16, we would take 56 and subtract 4k, or 16, from it. The amount 56 exceeds 16 (or 4k) would be 56 - 16, or 56 - 4k, choice c. If you have trouble with these types of questions, we strongly recommend using this procedure.

Another type of exam question will give the value of each of the letters in the algebraic expression, and ask for an evaluation of the expression. This is consistent, since letters in algebra simply represent unknown numbers. To answer these correctly, you need to substitute numbers for letters, keeping in mind the principles we described earlier.

1. Example: What is the value of the expression y^3-x^2, if x = -4 and y = -6?

a. 200 C. -42
b. 24 d. -232

It's less confusing if you place the number you are substituting for a letter in parentheses, and then continue. $(-4^{)2}$ would equal +16, and $(-6)^3$ would equal -216 (-6 times -6 times -6). So, -216 - (+16) would equal-232.

2. What is the value of $\frac{24m^4n^2}{6m2n3}$ if *m* = 3 and n = -4?

a. +16 c. +9
b. -9 d. -6

1t[1]s very helpful to first simplify something like this.

$\frac{24m^4n^2}{6m^2n^3}$ can be simplified. If you need a review:

$$\frac{(6)(4)(m)(m)(m)(m)(n)(n)}{(6)(m)(m)(n)(n)(n)} = \frac{\overset{1}{(\not 6)}(4)\overset{1}{(\not m)}\overset{1}{(\not m)}(m)(m)\overset{1}{(\not n)}\overset{1}{(\not n)}}{\underset{1}{(\not 6)}\underset{1}{(\not m)}\underset{1}{(\not m)}\underset{1}{(\not n)}\underset{1}{(\not n)}(n)} = \frac{4m^2}{n}.$$

Substituting in numbers, we would get $\frac{4(3)^2}{-4} = \frac{4(9)}{-4} = \frac{36}{-4} = -9.$

If you didn't simplify, and went ahead and multiplied and divided all those numbers, you would have ended up with something like 31,104 ÷ -3456 = -9. Simplifying saves a lot of time.

3. What is the value of $rs + \frac{x}{y}$, if r = 2, s = 4, x = 3, and y = 17?

   a. 8 3/17 c. 31 3/17

   b. 6 3/17 d. 24 3/17

   Substituting, $rs + \frac{x}{y} = (2)(4) + \frac{3}{17} = 8 + \frac{3}{17} = 8\ 3/17$

4. What is the value of k^2x^2, where k = 8 and x = -4?

   a. 256 c. 1024

   b. -128 d. 512

   k^2x^2 would equal $(8)^2(-4)^2 = (64)(16) = 1024$

5. What is the value of a(b+c+2d) + b, where a = 4, b = -3, c = 8, and d = 7?

   a. 79 c. 76

   b. 73 d. 18

   a(b+c+2d) + b = 4(-3+8+2(7)) + -3. We need to work first on the numbers inside of the parentheses. -3+8+14 = 19. Next we would multiply 4 x 19 = 76, and then subtract 3. 76 - 3 = 73.

6. What is the value of $\frac{(a+b+c)^2}{y-z}$, where a = -6, b = 7, c = 4, y = 2, and z = -3?

   a. -25 c. 5

   b. 25 d. -5

   $$\frac{(a+b+c)^2}{y-z} = \frac{(-6+7+4)^2}{2-(-3)} = \frac{5^2}{5} = \frac{25}{5} = 5$$

7. $N = 1/2r(x+y)$. The value of N if $r = -6$, $x = 5$ and $y = 12$ is:

a. 86 c. 102

b. -102 d. -51

It's a little different format, but the same procedure.
$N = 1/2r(x+y)$ equals $1/2(-6)(5+12)$. This equals $1/2(-6)(17) =$
$(-3)(17) = -51$, or $\frac{(-6)(17)}{2} = -51$.

We'll give you a few more to try. The answers are in the Answer Key.

21. Find R if $R = 192 + z^2$ and $z = -8$ and $b = 4$.

a. .50 c. .8125

b. 4 d. 1.50

22. How much simple interest is earned on a savings account of $1800 (p = 1800) at the rate of 61/2% (r = .065) for 3 years (t = 3)? Use the formula Interest = prt.

a. $351 c. $312

b. $247 d. $386

23. How far would a train travel at a rate of 85 mph (r = 85) in 12 hours (t = 12)? Use the formula distance = rt.

a. 610 miles c. 980 miles

b. 510 miles d. 1020 miles

24. Find A if $A = \frac{z^3 + y^2}{b^4}$, and $z = -5$, $y = 12$ and $b = 2$.

a. 4.87 c. 16.8

b. 1.19 d. 2.16

EQUATIONS

An equation is a statement that two expressions are equal. Knowing how to solve equations makes solving some exam problems easier. An equation can be distinguished from something. that is not an equation because an equation <u>always</u> has an equal sign between the two quantities being compared.

Examples: $4x + 17 = 29$

Area = length x width

$\frac{x}{4} + \frac{x}{6} = 1$

8 + 16 =24

The basic idea behind an equation is that an equation is a statement of equality between two quantities. In the examples above, the equal sign shows that whatever is on the left side of the equal sign must equal what is on the right side of the equal sign.

8 + 14 = 22

In the above example, you would have to have 22 be on the other side of the equal sign, or the equation would be a false equation. For example, 8 + 14 = 27 would be a false equation, while the above example is a true equation.

In algebra, equations are extremely valuable because they are used to solve for unknown numbers, and this in turn is very valuable for solving some promotional exam test questions.

In an algebraic equation, a letter is used to represent the unknown number we're trying to find. Then, by using the principles described below, we solve the equation by finding out what number the letter represents.

For example: $4x + 17 = 29$

We need to find what number x represents. What number, when you multiply it by 4, and then add 17 to it, will give you 29? Or, "4 times what number, plus 17 equals 29?". By experimenting and substituting in different numbers for x, we can find what number x represents. For example, let's start with the number one, and substitute that for x. Does (4 times 1) plus 17 = 29? 4 + 17 = 21, not 29. Substituting in 2 for x: Does (4 times 2) + 17 equal 29?
8 + 17 = 25, not 29. Substituting in 3 for x: Does (4 times 3) + 17 equal 29?
12 + 17 = 29, 50 x equals 3. Now, obviously, one wouldn't want to go through this for every equation, and a lot of times you wouldn't get numbers that would come out so evenly, so you'd need a better way to solve equations.

The Better Way

To find the solution of an equation, you should first remember that if the same expression is added or subtracted from both sides of an equation, the resulting expressions are still equal to each other.

Examples: 4 + 6 = 10. Let's add 5 to each side: 5+4+6 = 10+5. Or, we could use the above example, $4x + 17 = 29$. Adding 7 to each side (to illustrate this principle) would not change the value of x, it would still equal 3.
$4x + 17 + 7 = 29 + 7$. $4x + 24 = 36$. $4(3) + 24 = 36$. $12 + 24 = 36$. $36 = 36$.

The same would hold true for subtraction:

105 + 17 = 122 Subtracting 16, for example, from each side:

105 + 17 - 16 = 122 - 16

106 = 106

Or, using the 4x + 17 = 29 example:

4x + 17 = 29

4x + 17 - 16 = 29 - 16

4(3) + 17 - 16 = 13

12 + 17 - 16 = 13

29 - 16= 13

13 = 13

You also could add or subtract unknown variables (letters like x) from both sides, and the resulting equation would still be equal. For example:

12x + 14x = 26x. Let's subtract 5x from each side.

12x + 14x - 5x = 26x - 5x

26x - 5x = 26x - 5x

21x = 21x

The next principle to remember is that IF BOTH SIDES OF AN EQUATION ARE MULTIPLIED OR DIVIDED BY THE SAME EXPRESSION, THE RESULTING EXPRESSIONS ARE STILL EQUAL TO EACH OTHER.

Again, we can use the above example 4x + 17 = 29 to illustrate this.
4x + 17 = 297 50 x equals 3. Multiply each side of the equation by 2:

2(4x+17) = 2(29)

8x+ 34 = 58

8(3)+ 34 = 58

24 + 34 = 58

Or: 4x + 17 = 29. Divide each side by 3.

$$\frac{4x + 17}{3} = \frac{29}{3}$$

$$\frac{4(3) + 17}{3} = \frac{29}{3}$$

$$\frac{12 + 17}{3} = \frac{29}{3}$$

$$\frac{29}{3} = \frac{29}{3}$$

The kinds of equations used on promotional exams are linear equations, where there is only one unknown, and where the exponent of the unknown number is no larger than 1. (For example, no

x^2 or $x^3 = 584$, and no $3x + 7y + 8y = 451$, etc. If an x^2 or z^4 or something like that is included, they will be on opposite sides of the equal sign, and will eventually cancel each other out).

In order to solve linear equations:

FIRST, USING THE PRINCIPLES OUTLINED ABOVE, GET ALL THE UNKNOWNS (THE LETTERS) ON ONE SIDE OF THE EQUATION, AND ALL THE KNOWNS (THE NUMBERS) ON THE OTHER SIDE.

THEN, IF YOU HAVE LIKE TERMS, COMBINE THEM (see previous section on this if you want a review).

FINALLY, AGAIN USING THE PRINCIPLES OUTLINED ABOVE, ISOLATE THE UNKNOWN. In other words, if you have something like 14R on one side, you'll want to get rid of the 14 and have just the R. You'll then be able to solve the equation. The following examples will make this clearer.

Examples: $6x = 234$

1st step: Get all the numbers on one side, and the letters on the other.

This is already accomplished.

2nd step: Combine like terms. This is also already accomplished.

3rd step: Isolate the unknown. What we need to do here is turn 6x into just plain x, so we can find the answer. The above principles state that we can multiply or divide on either side of the equation by any number or expression, as long as we do the same operation to both sides of the equation. So, 6x would become x, or lx, if we divided it by 6. But what we do to one side of an equation we must do to the other side as well. If we divide one side by 6, we have to divide the other side by 6.

$$6x = 234$$

$$\frac{6x}{6} = \frac{234}{6} \qquad x = 39.$$

In algebra, it's critical that you check your answer by substituting the answer back into the equation in place of the letter. If both sides are equal to each other, your answer is correct.

More examples: $9z = 144$

$$\frac{9z}{9} = \frac{144}{9} \qquad z = 16$$

$$13z = 182 \qquad \frac{13z}{13} = \frac{182}{13} \qquad z = 14$$

The following examples require you to also use step 2, combining like terms.

$4a + 17a = 147$

$21a = 147$

$$\frac{21a}{21} = \frac{147}{21}$$

$a= 7$

$21r + 17r = 342$

$38r = 342$

$$\frac{38r}{38} = \frac{342}{38}$$

$r=9$

The following examples require you to use all three stages:

$13y + 17 = 6y + 38$

The first step is to get all of the letters on one side of the equation, and all of the numbers on the other side. Using the principles outlined earlier, we can do this by subtracting any quantity we wish from either side, <u>as long as we do the same thing to both sides</u>. Here, to get rid of the 17 on the left side, so we have just y's (or unknowns) there, we can subtract 17 from itself, to get zero. But, <u>we'll need to do the same on the other side</u>. So we'll need to subtract 17 from both sides.

$13y + 17 = 6y + 38$
$13y + 17 - 17 = 6y + 38 - 17$
$13y = 6y + 21$

Next, we have to get rid of the 6y on the right side. Again, if we subtract 6y from itself, we'll have zero y's on the right side. But we'll also have to do the same thing on the other side of the equation.

$13y = 6y + 21$
$13y - 6y = 6y - 6y + 21$
$$\frac{7y}{7} = \frac{21}{7}$$
$y=3$

There are other ways to do this. Some people mentally take a shortcut and bring the numbers and letters they want to get rid of "across the equal sign," changing signs as they do this. For example:

$$13y + 17 = 6y + 38$$

They would bring the 6y over to the side of the 13y. Since the 6y is "crossing the equal sign," it changes signs and becomes -6y. So we now have

$$13y + 17 - 6y = 38$$

Then they would take the 17, and bring it over to the "side with the numbers." But it too, because it is crossing the equal sign, would change signs and become -17.

$$13y - 6y = 38 - 17$$

Now we have letters on one side and numbers on the other.

$$7y = 21$$

$$y=3$$

This method is a simplification of the first method we described. The reason it works is that it is based on the same principles as the first method, it's just a shortcut. You should use whatever method you're most comfortable with. Here are a few more examples, which we'll do the non-shortcut way:

1. $24 + 14a = 6a + 128$

$$24 + 14a - 24 = 6a + 128 - 24$$

$$14a = 6a + 104$$

$$14a - 6a = 6a - 6a + 104$$

$$8a = 104$$

$$\frac{8a}{8} = \frac{104}{8}$$

$$a = 13$$

We should then check it to see if the answer is correct, by substituting in 13 for a.

$$24 + 14a = 6a + 128$$

$$24 + 14(13) = 6(13) + 128$$

$$24 + 182 = 78 + 128$$

$$206 = 206$$

2. $7x + 35 = 2x + 118$

$$7x + 35 - 35 = 2x + 118 - 35$$

$$7x = 2x + 83$$

$$7x - 2x = 2x - 2x + 83$$

$$5x = 83$$

$$\frac{5x}{5} = \frac{83}{5}$$

$$x = 16.6$$

Checking it: $7(16.6) + 35 = 2(16.6) + 118$

$116.2 + 35 = 33.2 + 118$

$151.2 = 151.2$

3. $15x - 47 = 4x + 108$

(In this case, to get rid of the -47, we[1]ll need to add +47 to it, to make it equal zero. That means we'll have to add 47 to the other side.)

$$
\begin{aligned}
15x - 47 &= 4x + 108 \\
15x - 47 + 47 &= 4x + 108 + 47 \\
15x &= 4x + 155 \\
15x - 4x &= 4x - 4x + 155 \\
11x &= 155 \\
\frac{11x}{11} &= \frac{155}{11}
\end{aligned}
$$

$x = 14.\overline{09}$ (The bar over the .09 means the .09 repeats continuously).

Checking it: $15x - 47 = 4x + 108$

$15(14.09) - 47 = 4(14.09) + 108$

$211.35 - 47 = 56.36 + 108$

$164.35 = 164.36$

(Because the .09 is a repeating decimal, the numbers here will be off a little).

4.

$$
\begin{aligned}
4x + 13 &= 8x + 46 \\
4x + 13 - 13 &= 8x + 46 - 13 \\
4x &= 8x + 33 \\
4x - 8x &= 8x - 8x + 33 \\
-4x &= 33 \\
\frac{-4x}{4} &= \frac{33}{4}
\end{aligned}
$$

$x = -8.25$ (For a review of negative and positive numbers, see the section above on signed numbers).

Checking it: $4(-8.25) + 13 = 8(-8.25) + 46$

$-33 + 13 = -66 + 46$

$-20 = -20$

5. $275r + 64 = 15r + 869$

$$275r + 64 - 64 = 15r + 869 - 64$$
$$275r = l5r + 805$$
$$275r - l5r = 15r - l5r + 805$$
$$260r = 805$$
$$\frac{260r}{260} = \frac{805}{260}$$

$r = 3.0961$ (approximately)

Checking it:
$$275(3.096) + 64 = 15(3.096) + 869$$
$$851.4 + 64 = 46.44 + 869$$
$$915.4 = 915.4$$

(For most exam purposes, you could round off the 3.096 to 3.10).

6. $-5x + 24 = 3x - 48$

$$-5x + 24 - 24 = 3x - 48 - 24$$
$$-5x = 3x - 72$$
$$-5x - 3x = 3x - 3x - 72$$
$$-8x = -72$$
$$x=9$$

Checking it:
$$-5(9) + 24 = 3(9) - 48$$
$$-45 + 24 = 27 - 48$$
$$-21 = -21$$

(Again, see section on signed numbers if you need a review of working with positive and negative numbers).

7. $14m + 16 = 8m + 480$

$$14m + 16 - 16 = 8m + 480 - 16$$
$$14m = 8m + 464$$
$$14m - 8m = 8m - 8m + 464$$
$$6m = 464$$
$$\frac{6m}{6} = \frac{464}{6}$$
$$m = 77.33$$

Checking it:
$$14(77.33) + 16 = 8(77.33) + 480$$
$$1082.666 + 16 = 618.666 + 480$$
$$1098.666 = 1098.666$$

This knowledge of linear equations is what is required to solve word problems on promotional examinations. What follows are some word problems for you to practice with. Explanations follow the questions.

WORD PROBLEMS

25. The formula for the perimeter of a rectangle is $P = 2l + 2w$. If the perimeter of a certain rectangle is 75 cm and its length is 28 cm, what is the measure of its width?

    a. 9.5 cm c. 25 cm
    b. 19 cm d. 12.5 cm

26. Three times a number plus fifteen is equal to seventeen minus the number. The number is:

    a. 1 c. 1/2
    b. -l d. -1/2

27. Fourteen times a number minus seventy is equal to four times the number minus fifteen. The number is:

    a. 6.8 c. 4.7
    b. 5.5 d. 5.9

28. The velocity of a falling object at any time (v_t) with initial velocity v_0 is given by the equation $v_t = v_0 + 32.\ 2t$. What is the initial velocity if the velocity after 4 seconds is 196 feet per second?

    a. 67.2 feet/second c. 163.8 feet/second
    b. 33.6 feet/second d. 134.4 feet/second

29. The best representation of the sum of the squares of a and 4 would be:

    a. 2(a + 4) c. (a + 4)2
    b. (a + 4)3 d. $a^2 + 16$

30. The best representation for forty-four percent of twelve more than R would be:

    a. 44(R + 12) c. .44(12) + R
    b. 44(R) + 12 d. .44(R + 12)

31. Pat received a score of 84 on the written portion of a promotional examination. What score must Pat receive in the evaluation of her training and experience to receive a score of 90? (The two scores are averaged to find the final score).

    a. 94 c 96
    b. 98 d. Can not be determined from information given

32. Jane can paddle a canoe 3 miles an hour in still water. If she paddles with the current, she can go 30 miles in 6 hours. How fast is the current? (distance = rate x time)

a. 1.5 miles per hour	c. 5 miles per hour
b. 2 miles per hour	d. 1.6 miles per hour

33. A typewriter and a dictation machine cost a total of $840. If the typewriter cost $360 more than the dictation machine, how much did the dictation machine cost?

a. $480	c. $240
b. $440	d. $280

34. If Janet can build 22 tables in 14 days, and Anne can build 22 tables in 16 days, approximately how long will it take them to build 22 tables together?

a. 9.5 days	c. 15 days
b. 7.5 days	d. 8 days

35. If a number is multiplied by fourteen, and then increased by seven, the result is 141. The number is approximately:

a. 9.6	c. 10.1
b. 8.8	d. 9.4

EXPLANATIONS FOR WORD PROBLEMS

25. This question involves substituting the numbers given, and then solving the equation to find the width, w. We are told P = 2*l* + 2w, and that P = 25 and *1* = 28. So:

$$75 = 2(28) + 2w$$
$$75 = 56 + 2w$$

$$75 - 56 = 56 + 2w - 56$$
$$19 = 2w$$
$$\frac{19}{2} = \frac{2w}{2}$$
$$9.5 = w$$

(If you wanted to write it as 2*l* + 2w = P, since that would put the unknown on the left side of the equation, you could do that. It doesn't matter[1] as long as numbers are on one side and letters are on the other.)

Checking it:

$$P = 2l + 2w$$
$$75 = 2(28) + 2(9.5)$$
$$75 = 56 + 19$$
$$75 = 75$$

26. It helps here to substitute in a letter, like x, whenever they use the phrase "a number", If you're not sure how to set this up, you should review the section on algebraic expressions that involves setting up expressions correctly. Substituting in x for the phrase, "the number", we will get:

3 times x + 15 = 17 - x

Please note that the problem reads "17 minus the number", so it is 17-x. If it had w said "the number minus 17", it would be written x - 17. Also note that up incorrectly as x - 17, choice b was there to falsely reassure you.

So,

$$3x + 15 = 17 - x$$
$$3x + 15 - 15 = 17 - x - 15$$
$$3x = 17 - x - 15$$
$$3x = -x + 2$$
$$3x - (-x) = -x + 2 + x$$
$$4x = 2$$
$$x = 1/2$$

Checking it:

$$3(1/2) + 15 = 17 - 1/2$$
$$1.5 + 15 = 16.5$$
$$16.5 = 16.5$$

27. Again, setting this up correctly is very important.

$$14x - 70 = 4x - 15$$
$$14x - 70 + 70 = 4x - 15 + 70$$
$$14x = 4x + 55$$
$$14x - 4x = 4x + 55 - 4x$$
$$10x = 55$$
$$x = 5.5$$

Checking it: $14(5.5) - 70 = 4(5.5) - 15$

$$77 - 70 = 22 - 15$$
$$7 = 7$$

Again, if you had set up the relationship between the numbers incorrectly, choice c would have falsely reassured you.

28. We are told that $v_t = v_0 + 32.2t$, and that the velocity after 4 seconds is
second. Don't be intimidated by this question. Once again, it's
numbers for letters and then solving the equation.

$$v_t = v_0 + 32.2t$$
$$v_t \quad 196 \text{ feet per second}$$
$$v_0 = X$$
$$t = 4 \text{ seconds}$$

So,

$$196 = v_0 + 32.2(4)$$
$$196 = v_0 + 128.8$$
$$196 - 128.8 = v_0 + 128.8 - 128.8$$
$$67.2 = v_0$$

29. This is pretty tricky for most people. If you missed it, you may want to review the section on multinomial numbers in algebraic expressions. Choice a is incorrect because this does not represent the squares of a and 4, this represents two times a and 4 (see section on exponents). Choice b is incorrect because this is the cube of (a + 4). Choice c is incorrect because this is the square of the sum of a and 4, not the sum of the squares of a and 4. Choice c would be $a^2 + 8a + 16$ (see section on multiplying multinomial expressions). Choice d is correct. The square of a is a^2 and the square of 4 is 16. Their sum would be $a^2 + (4)^2$, or $a^2 + 16$.

30. .44(R + 12) is the only correct representation of the four choices. If you missed this, we recommend reviewing the section on multiplying algebraic expressions. a is incorrect because 44% must be expressed as .44 in this situation.

31. As we explained in Booklet 2 of this series, you can often work backwards from the answers given to find the correct choice, and you could do that to solve this problem. You could add each of the possible choices to 84, and then divide by 2 to see if you got 90. Algebra is, however, faster to use here. The way you would set this up would be like this:

$$\frac{x + 84}{2} = 90$$

What number, when averaged with 84, would give you 90?

$$\frac{x+84}{2} = 90$$

Using principles discussed earlier, we can multiply both sides of the equation by two to get rid of the fraction.

$$2 \text{ times } \left(\frac{x + 84}{2}\right) = 90 \text{ times } 2$$

$$x + 84 = 180$$
$$x + 84 - 84 = 180 - 84$$
$$x = 96$$

You can check it: $\frac{96 + 84}{2} = 90$

$$\frac{180}{2} = 90$$
$$90 = 90$$

32. We're told distance = rate x time, and we need to find the speed of the current. The best way to do this would be to let x equal the speed of the current. Then x + 3 (3 is the number of miles Jane can row in a::' hour) would be Jane's speed when the current is helping her. (x, the current, plus Jane's speed without the current, 3, would equal x + 3, Jane's speed <u>with</u> the current). So x + 3 is the rate. We're told the distance she travels is 30 miles in 6 hours with the current. D = rt, so 30 = (x + 3)6, 30 = 6x + 18

$$30 - 18 = 6x + 18 - 18$$
$$12 = 6x$$
$$\frac{12}{6} \quad \frac{6x}{6}$$
$$2 = x$$

So the current is 2 miles per hour. A problem like this is difficult for many people. Once people realize they can always do these by letting x equal the speed of the current, and letting x + whatever the person' 5 rate is equal the rate when the current is helping the person, they can solve these much more easily.

33. This is the same problem as Question 8 in Booklet 2. We’ve includ demonstrate how quickly it can be solved by using algebra. We cou the dictation machine equal x. Since the typewriter is $360 more machine, it would equal x + 360. Together their cost is 840. So:

$$
\begin{aligned}
x + (x + 360) &= 840 \\
2x + 360 &= 840 \\
2x + 360 - 360 &= 840 - 360 \\
2x &= 480 \\
x &= 240
\end{aligned}
$$

34. Again, another problem from Booklet #2. In that booklet, we demonstrated a procedure you can memorize that will always work to solve these work problems. But we'd like you to also see how it can be done algebraically.

Let x equal the time the task takes. Janet can build 22 tables in build 22 tables in 16 days. In x days, Janet does x/14 part of the work and Anne does x/ part of the work. Together they complete the job. Here, you can have the complete job represented by 1. So,

$$\frac{x}{14} \quad \frac{x}{16} = 1$$

(Multiply the equation by the lowest common denominator, 112, to get rid of the fractions).

$$
\begin{aligned}
8x + 7x &= 112 \\
15x &= 112 \\
x &= 7.466
\end{aligned}
$$

A slight variation is to let x equal the time it takes them to do the task. If it takes x days to do a job, 1/x of the job is done each day. Janet would do 1/14 of the job in one day, and Anne would do 1/16 of the job in one day. We could then let

$$\frac{1}{x} = \frac{1}{16} + \frac{1}{14}$$

Multiplying by 224x, to get rid of the fractions, we would get

$$
\begin{aligned}
224 &= 16x + 14x \\
224 &= 30x \\
7.466 &= x
\end{aligned}
$$

This is very difficult for most people to remember or set up correctly, which is why we've also included the following approach, which is based on these algebraic principles, and *always* works, but is easier for almost everyone.

1) first invert the 2 numbers you're given
2) then add them together
3) finally, invert again

Don't feel badly if you can't quite visualize the algebra involved. We've included it mainly as a point of interest. As long as you remember the three steps up above, you'll always get the correct answer.

35. Let x represent the phrase "a number". If x is multiplied by fourteen, and then increased by seven, the result is 141. So:

$14x + 7 = 141$

$14x + 7 - 7 = 141 - 7$

$14x = 134$

$\frac{14x}{14} = \frac{134}{14}$

x = 9~57

Checking it: $14(9.57) + 7 = 141$

$133.98 + 7 = 141$

$140.98 = 141$

(approxmiately)

This concludes our section on algebra. We've provided you with the basics needed for promotional exams. Good luck!

ANSWER KEY

1. -4
2. -2
3. -46
4. 19
5. -4
6. 40
7. 144
8. -98
9. -12
10. 13
11. 2m-4
12. $\frac{2}{y^2} - \frac{3x}{7y}$
13. 3m+17
14. 19-4m
15. $\frac{m}{8}$ or l/8m
    8
16. m-14+3m
17. 4r-7s
18. rs+3r
19. $\frac{r^2+s}{2}$ or $r^2+1/2s$
20. $\frac{2s^2+r}{3}$ or $2s^2+l/3r$
21. b
22. a
23. d
24. b
25. a
26. c
27. b
28. a
29. d
30. d
31. c
32. b
33. c
34. b
35. a

SOME NOTES ON STATISTICS FOR THE SENIOR STATISTICS CLERK EXAM

While it is impossible to predict exactly what algebraic and statistical operations will be on this exam, there are certain operations that traditionally have been required. The exam announcement states that the exam will include "Arithmetic reasoning and algebra; and elementary statistical methods and computations." Traditionally there have been 15-20 multiple choice questions on this section. There are usually a number of algebraic questions that require you to solve for x (or R, or p, or m). For example: 4x+8 = 15. Solve for x. If you've reviewed the preceding pages, you should have no trouble with these types of questions.

$$4x + 8 = 15$$
$$4x + 8 - 8 = 15 - 8$$
$$4x = 7$$

$$\frac{4x}{4} = \frac{7}{4}$$

$$x = 1.75$$

Checking it:

$$4(1.75) + 8 = 15$$
$$7 + 8 = 15$$
$$15 = 15$$

Of course there will probably be more difficult algebraic questions as well, but if you understand the examples and problems given in the preceding pages you shouldn't have much trouble with these. Sometimes there are one or two tables where they will give you some data, and you are required to answer questions about the data. This traditionally has required knowledge of percent increase and decrease (see below or Booklet 3 on tabular material if you're unsure of how to do these), and good overall math skills. They may give you a table and a statistical formula, and require you to use your math and algebraic skills to find the answer.

An example of a percent increase and decrease problem: Suppose you were given a table that showed that the number of people interested in statistics in Agency Y in 1982 was 412. In 1983, the number of people interested in statistics in Agency Y was 427. You might then be asked to find the increase between the two years, in terms of percents.

The procedure for finding percent increase or decrease is always the same.

1) TAKE THE DIFFERENCE BETWEEN THE TWO NUMBERS BEING COMPARED
2) THEN DIVIDE THIS DIFFERENCE BY THE NUMBER THAT CHRONOLOGICALLY CAME FIRST IN THE TABLE

$$427 - 412 = 15$$

$$\frac{15}{412} = .0364 = 3.64\% \text{ increase}$$

While not much actual working knowledge of statistics has been requir will usually give you statistical formulas and, using your knowledge will be required to substitute in numbers for letters and then se sometimes definitions of common statistical terms, and methods for required.

Elementary Statistical Terms and Methods

By definition a population is an entire group of people, objects or events all having at least one characteristic in common. In statistics, the term refers to the entire group of individuals or events to which generalizations are to be made. A population may vary in size fro agency to several million persons or events or even to an infinite measures, such as an average, are obtained for these defined popul parameters. A parameter is a numerical characteristic of a populatio the average age of all of the employees in Tax and Finance, the averag that population.

Because of the large size of most populations, it usually is not possi element in that population. Nielson ratings, for example, are used because it woul expensive and impractical to measure what television shows are being watched in each household in the U.S. It is usually sufficient to select and measure what is known as a sample of the persons or objects, and then make inferences about the entire population on the basis of the sample data. A sample is defined as a subgroup selected from the complete population. However, if sample data are to be used as the basis for generalizations about the population, it is essential that the sample be truly representative of that population.

Measures obtained from a sample, such as average promotional examination scores or political opinion polls, are called statistics. A statistic is any numerical characteristic of a sample. It indicates something about a sample as a parameter indicates something about a population. Lower case italic letters are usually used to symbolize statistics and lower case Greek letters are used to symbolize parameters. Statistics may also be used with populations.

One of the most efficient ways to summarize and organize data is to arrange them into what is called a frequency distribution. A frequency distribution is an organization of measures or observations into score classes along with the frequency of occurrence in each class. For example, a table that listed the previous exam scores of Agency employees would be cumbersome to read. However, if a frequency distribution were used, a great deal more sense could be made of the table, as it would condense the information, and reveal how many employees received each score. In preparing a frequency distribution, we first list all the possible scores and then go through the collection of scores, making a tally mark next to each score in the list every time it occurs. To prepare the final form we simply total the tally marks to show the frequency of occurrence of each score.

We often need more concise methods of summarizing information than frequency distribution, however. There are three commonly used ways to convey information about the average scores in a distribution, and each interprets average in a slightly different way. They are called mode, median and mean, and are referred to as measures of central tendency.

The mode is defined as the score value which occurs most frequently in a group of scores. We identify the mode by looking at the scores and locating the one which occurs most often. Examine the following scores: 107, 108, 108, 108, 109, 111, 111, 123, 123, 123, 123, 125. 123 is the mode, as it occurs most often. The most frequently occurring score is usually somewhere near the center of a distribution. In such a case, the mode is a legitimate index of central tendency. However, as in this case, the mode does not always occur near the center of a distribution, and therefore we can't rely on it to accurately reflect the center of a set of scores. Because of its instability the mode is seldom used to indicate central tendency. Its use is largely restricted to that of a quick inspection kind of "average".

The median is the middle point of a distribution. Half of the distribution is located above the median, and half is below the median. The first step in locating the median is to arrange the scores in order, enabling us to locate the middle point. Consider the following scores: 63, 68, 72, 78, 79, 81, 85. The median is 78 since it is the middle score'. there are three scores above the median 78, and three scores below it. If the total number of scores is even, the median is the score that is the upper real limit of the lower of the two middle scores, and the lower limit of the higher score. For example, if we add a score of 90 to the above numbers, we get 63, 68, 72, 78, 79, 81, 85, 90; the median is then 78.5. The median is the point that separates the real limits of the middle scores 78 and 79.

When there is a repetition of the same score in the middle of the distribution, the location of the median requires computation. For example: 84, 85, 86, 87, 87, 87, 88, 89, 90, 90. This particular type of computation is known as interpolation. It is discussed in detail in all introductory statistics textbooks. But you should not need to know interpolation for this exam.

The mean is the best known and most reliable measure of central tendency The mean is the arithmetic average of a group of scores. It is found by adding all the scores in the distribution and then dividing the sum of the scores by N, the total number of scores.

$$\text{Mean} = \frac{\text{Sum of Scores}}{N}$$

In statistics, the capital Greek letter Σ (sigma) means '1the sum of." The expression Σ X would tell you to "sum the x's (the scores)." Again the mean is equal to the sum of the scores divided by the number of scores.

$$\text{Mean} = \frac{\Sigma N}{N}$$

The use of the mean has many advantages. It is sensitive to all of the scores in a distribution, which is usually desirable. In the case of extreme scores, however, the use of a mean can be a disadvantage (remember in high school how a few abnormally high test scores would affect the class average?). Before we continue on, we thought it would be helpful to include a review of square roots, as sometimes they are included on the exam.

Square Roots

It's helpful for some formulas to know how to work with square roots. A perfect square is a whole number or fraction that is the square of another whole number or fraction. Examples: $9 = 3^2$ and $1/25 = 1/5^2$, so 9 and 1/25 are perfect squares. The square root of a positive number is squared to give the number. The symbol √ shows the square root of a number. $\sqrt{9} = 3$ because $3^2 = 3 \times 3 = 9$.

$\sqrt{1/25} = 1/5$ be cause $(1/5)^2 = 1/5 \times 1/5 = 1/25$.

The whole number 49 is the square of 7, so 7 is the square root of 49.

$7^2 = 49$ or $\sqrt{49} = 7$

If the whole number involved is a perfect square, the square root can be found by using a calculator, or by trial and error. For example:

$\sqrt{324}$

You could use trial and error, or a calculator, to find that 18 x 18, or $18^2 = 324$.

Naturally, some numbers are not perfect squares. $\sqrt{108}$, for example, does not come out evenly, as it equals approximately 10.392. You may remember learning in school a procedure for finding square roots. Fortunately, the digital revolution has come along, and calculators are now used instead in most cases. For our exam purposes, if square roots do appear on an exam, either they will be perfect squares (they'll come out evenly), or, most commonly, they'll be left in square root form. For example, you'll have to choose between choices that might look like this:

a. $\sqrt{471}$ c. $\sqrt{927}$
b. $\sqrt{808}$ d. $\sqrt{1006}$

You should not be required to calculate square roots without a calculator. We've given you this review of square roots because they are sometimes involved in the formulas you are given, as we'll soon see.

Substituting Numbers for Letters In Statistical Formulas

On this exam you are sometimes required to substitute numbers for letters in statistical formulas. You are not usually expected to have memorized these formulas. You are expected to have the algebraic and math skills necessary to successfully solve these operations. Please see previous section for practice in substituting values (numbers for letters) in algebra if you're not sure how to do this. However, occasionally there have been three methods tested where it would have been helpful if candidates had some previous exposure to the methods. These are finding the mean, variance, and standard deviation.

MEAN, VARIANCE, STANDARD DEVIATION

An example of a question involving finding the mean:

1. The formula for calculating the sample arithmetic mean is $\bar{X} = \frac{\Sigma X}{N}$

where $\bar{X}$ is the sample mean, X is the score, ΣX is the sum of the X scores, and N is the number of cases in the sample. The scores are 96, 81, 88, 75, 70 and 80. The sample mean would be:

a. 79.78 c. 81.67
b. 82.45 d. 80.56

This looks a lot more difficult than it is. In this case, you can answer this successfully by just substituting in numbers. Here, $\bar{X}$ (pronounced X bar, but you won't need to know that), equals ΣX/N , and we need to find $\bar{X}$. We're told that ΣX is the sum of the X scores, and N is the number of cases in the sample. The scores are 96, 81, 88, 75, 70 and 80. Since ΣX equals the sum of the scores, we should first add the scores together to find ΣX. 96+81+88+75+70+80 = 490. N = number of cases in the sample. This confuses some people. It means the number of cases, or in this case, the number of scores you are considering. Here we are considering 6 scores.

$\bar{X} = \frac{\Sigma X}{N}$, so all we have left to do is substitute in numbers for each of the letters. $\bar{X} = \frac{490}{6} = 81.\overline{66}$ (the bar over the .66 means repeating decimal)

Basically, what you've done here is taken a simple average. The formula makes it look a lot more intimidating. If you'd like more practice in substituting numbers for letters, see the algebra section.

Variance

Sometimes measures of variability are given on the exam. While formulas are almost always provided, it may be helpful to have some background information on this to make things clearer. A measure of variability indicates how much spread or dispersion there is in the scores. Distributions of scores may have the same mean yet differ in the extent of variation of the scores. Information about variability may be as important or more important than information concerning central tendency. For example:

Test Scores In Unit A	Test Scores In Unit B
90	95
90	70
85	75
85	75
40	75
390	390

390 ÷ 5 = a mean of 78	390 ÷ 5 = a mean of 78

The two sets of scores have the same mean, yet reflect important differences. In statistics, they have derived an index based on how each score deviates from the mean score, so that they will have a more stable index of the variability in a distribution. These measures of variability are called variance and standard deviation. Sometimes on exams you are given a group of numbers, or a table, and the formula, and asked to find variance and standard deviation. This becomes a lot easier if you can remember that <u>at the heart of variability is the need to consider the extent to which each individual score deviates from the mean of that distribution. This means you subtract the mean from each score to determine the deviation of the score from the mean</u>. This is expressed as $X - \mu = x$. X = score, μ (mu, pronounced meu) = mean, and x = deviation score.

For example:

Score
20
19
12
13
12
20

First we must find the mean. 20+19+12+13+12+20=96. There are 6 scores so we would divide 96 by 6 (mean = $\Sigma X/N$) = 16. The deviation for each score will be determined by its relationship to the mean, which is 16. For example, the score 20 is a deviation of +4 from 16.

So, the deviation would look like this:

Score (x)	Deviation Score (x)
20	+4
19	+3
12	-4
13	-3
12	-4
20	+4

The deviation scores (represented by x) provide a good basis for measuring the spread of scores in a distribution. However, because the sum of deviation scores in any distribution will add up to zero, it's not possible for statisticians to use the scores of deviations to get a useful index of the spread of the scores. To overcome this, they square all of the deviation scores. Then these squared deviations are added to give a measure called the sum of the squared deviation scores. Taking the squares of the earlier deviation scores:

Score (X)	Deviation Score (x)	Squared Deviation (x^2)
20	+4	16
19	+3	9
12	-4	16
13	-3	9
12	-4	16
20	+4	16

(Remember, squaring negative numbers makes them become positive numbers). The sum of the squared deviations in this case would be 16+9+16+9+16+16= 82.

Sometimes on exams you will be required to find the variance. Variance is derived from the deviation of scores from their mean. Variance is the mean of the squared deviation scores. To find variance, we would add the squared deviation scores and then divide by the number of scores. The population variance is represented by the lower case Greek letter sigma (σ) raised to the second power (squared), σ^2 So the formula for variance is:

$$\sigma^2 = \frac{\Sigma x^2}{N}$$

σ^2 = variance, Σx^2 = the sum of the squared deviations, and N = the number of scores.

In the above case, we've found that the sum of the squared deviations is 82, and we know there were 6 scores. So the variance would equal 82 ÷ 6 = l3.66. We substituted in the appropriate numbers. By finding the variance for different groups of scores, you can determine which groups of scores vary the most. The higher the variance, the greater the difference between the scores and the mean score.

To find the variance:

1) Find the mean of the group of numbers you are considering
2) Find the deviation from the mean for each score
3) Square each of these deviation scores
4) Find the sum of the squared deviations
5) Use the formula variance equals the sum of the squared numbers divided by the number of scores.

2. What is the variance of the following set of scores: 12, 7, 4, 6, 9, 10,13, 3?

a. 11.8 c. 13.4
b. 9.5 d. 12.7

We first need to find the mean for this group of numbers. 64 12+7+4+6+9
The total number of scores is 8, so we would divide 64 by 8. The mea

Then we need to find the deviation from the mean, which is 8, for each score.

Score	Deviation Score
12	+4
7	-1
4	-4
6	-2
9	+1
10	+2
13	+5
3	-5

The third step is to square each of these deviation scores.

Deviation Score	Squared Deviation
+4	16
-1	1
-4	16
-2	4
+1	1
+2	4
+5	25
-5	25

Next we need to find the sum of the squared deviations. 16+1+16+4+1+4+25+25=92. Finally, we use the formula variance equals the sum of the squared numbers divided by the number of scores, $\sigma^2 = \frac{\Sigma x^2}{N}$.

We know the sum of the numbers squared is 92, and that there are 8 scores. So be 92   8. The variance equals 11.5.

It's unlikely that you would not be given any clues how to do variance (they may give you the formula, or the text of the question may describe how to do it). We've included the above because we can't be sure they won't ask a question like the above. If you are very serious about doing well on the statistical clerk exam, we suggest you review the above questions again, and try and remember the steps involved, just in case. You could even make up your own questions for more practice, if you'd like. Of course it's very possible you may not need to know how to do variance without a formula, or even with a formula. But we've included it because we can't be sure, as there's no way to know in advance.

The same applies to the next measure of variability we're going to discuss, standard deviation. In the past, exams have sometimes included data from part of a table, and candidates have been required to find the standard deviation. Usually they include a formula, but occasionally they haven't. We've included the following just in case they don't include enough information on standard deviation for you to solve it on your own.

The Use of Standard Deviation from Raw Scores

The usefulness of variance is limited because in the process of squaring the deviation scores we departed from the original unit of measurement. We squared the scores, so the answer becomes a square answer. For example, if we'd been trying to find the variance of workers who participated in a certain training program, our unit of measurement using variance would end up being square workers. This can be remedied easily, however, by taking the square root of the variance. When we find the square root of the variance, we're finding the standard deviation. The formula for the standard deviation, when the mean is a whole number, is best represented by:

$$\sigma = \sqrt{\Sigma x^2 / N}$$

σ = (sigma) Standard Deviation, Σx^2 = the sum of the squared deviation scores, and N = the number of scores in the distribution. (On exams, the mean should always be a whole number for these types of questions. If not they would give you the formula).

In Question #2, $\Sigma x^2 = 92$, and N was 8. So the variance was $\sigma^2 = \frac{92}{8} = 11.5$.

The standard deviation would be the square root of $\frac{92}{8} = \sqrt{\frac{92}{8}} = \sqrt{11.5} =$ 3.39 (approximately). On an exam they would either leave the possible answers in square root form, have the answer be a perfect square, a readily identifiable square root, or give you a calculator or square root table.

It is also possible to compute the standard deviation directly from raw scores without first figuring out all of the deviation scores and squaring them. Of course if you're comfortable with this method, don't change it, as the exam questions on this traditionally don't give you lots of numbers to compute. We're including this for those of you who would like a shortcut, and also because it provides you with more practice working with formulas. Algebraically, it can be proven that the sum of the squared deviation scores is equal to the sum of the squared raw scores minus the sum of the raw scores squared divided by N.

$$\Sigma x^2 = \Sigma X^2 - [(\Sigma X)^2/N]$$

ΣX^2 means we have to square each raw score first and then sum those squares; the second value $(\Sigma X)^2$ requires us to sum the raw scores first and then square that sum. So another formula for finding standard deviation would be:

$$\sigma = \sqrt{\frac{\Sigma X^2 - \frac{(\Sigma X)^2}{N}}{N}}$$

Let's try this using Question #2 as an example. We need to find the sum of the squared raw scores, as ΣX^2 = the sum of the squared raw scores. (Note that we have to discriminate in this question between "the sum of the squared raw scores" and "the sum of the raw scores, squared". You may remember a similar type of distinction in Question 29 of the section on Algebra). So we would take each of the scores and square them.

X	X^2
12	144
7	49
4	16
6	36
9	81
10	100
13	169
3	9

The sum of all these X^2 is 604. So $\Sigma X^2 = 604$. Next we need to find $(\Sigma X)^2$, the sum of the raw scores, squared. The sum of the raw scores equals 12+7+4+6+9+10+13+3 = 64. We then need to square this number. $64^2 = 4096$.

$$\text{So, } \sigma = \sqrt{\frac{604 - \frac{4096}{8}}{8}}$$

$\Sigma X^2 = 604$
$(\Sigma X)^2 = 4096$
N = number of scores, 8

$$\sigma = \sqrt{\frac{604 - 512}{8}}$$

$$\sigma = \sqrt{\frac{92}{8}}$$

$$\sigma = \sqrt{11.5}$$

Some people don't find this to be a shortcut. It does provide more practice with formulas here, is sometimes given to candidates to work with on the exam, and can be used when the mean is not a whole number, so it can be very useful. Let's try one more.

3. The standard deviation of the following scores 1, 2, 3, 4, 5 would be:

   a. $\sqrt{2}$ c. $\sqrt{4}$

   b. $\sqrt{3}$ d. $\sqrt{2.5}$

X	X^2
1	1
2	4
3	9
4	16
5	25

The sum of X, (ΣX), would be 1+2+3+4+5= 15. The sum of X^2, $(\Sigma X)^2$, would be 1+4+9+16+25= 55.

$$\sigma = \sqrt{\frac{\Sigma X^2 - \frac{(\Sigma X)^2}{N}}{N}}$$

$$\sigma = \sqrt{\frac{55 - \frac{(15)^2}{5}}{5}}$$

$$\sigma = \sqrt{\frac{55 - \frac{225}{5}}{5}} = \sqrt{\frac{55 - 45}{5}} = \sqrt{\frac{10}{5}} = \sqrt{2}$$

As we stated at the beginning of this section, it's impossible to predict exactly what will be on an exam. We've tried in this booklet to provide a working knowledge of the methods that will most likely be required on exams of this nature. Of course the form the exam questions will take will be different, but the principles and skills involved will be similar. If you have worked conscientiously with this booklet, and understand the principles described, you should be able to apply these principles to the exam questions successfully.

Good luck!

We'd appreciate any comments or suggestions you may have about this booklet. Please send these to:

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